NARAYANA JUNIOR COLLEGE
AP EAMCET-2016
SET-A

MATHS

1. The domain of the function $f(x)=\sqrt{\log_{0.5} x!}$

1. $\{0,1,2,3,\ldots\}$ 2. $\{1,2,3,\ldots\}$ 3. $(0,\infty)$ 4. $\{0,1\}$

KEY: 4

HINT: $\log_{1/2} \angle x \geq 0$

$\Rightarrow \log_{1/2} \angle x \geq \log_{1/2} 1$

$\Rightarrow \angle x \leq 1$

$x=\{0,1\}$

2. If $f(x)=|x-1|+|x-2|+|x-3|, 2<x<3$**, then** f **is**

1. an onto function but not one-one 2. One-one function but not onto
3. a bijection 4. Neither one-one nor onto

KEY: 3

HINT: $2<x<3 \Rightarrow f(x)=(x-1)+(x-2)-(x-3)$

$=x-1+x-2-x+3=x$

3. The greatest positive integer which divides $(n+16)(n+17)(n+18)(n+19)$**, for all positive integers n, is**

1. 6 2. 24 3. 28 4. 20

KEY: 2

HINT: Product of four consecutive positive integers is divisible by $\underline{|4}$

4. If a,b,c are distinct positive real numbers, then the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ **is**

1. <0 2. >0 3. 0 4. ≥ 0

KEY: 1

HINT: $3abc-a^3-b^3-c^3=-\left(a^3+b^3+c^3-3abc\right)$

$=-(a+b+c)\left\{a^2+b^2+c^2-ab-bc-ca\right\}<0$

5. If x_1,x_2,x_3 **as well as** y_1,y_2,y_3 **are in geometric progression with the same common ratio, then the points** $(x_1,y_1),(x_2,y_2),(x_3,y_3)$ **are**

1. vertices of an equilateral triangle 2. vertices of a right angled triangle
3. vertices of a right angled isosceles triangle 4. Collinear

KEY: 4

HINT: $x_1,x_2,x_3=a,ar,ar^2$ are in G.P

$y_1,y_2,y_3=b,br,br^2$ are in G.P

Slope of line joining $(x_1,y_1)(x_2,y_2)$ = Slople of line joining (x_1,y_1) and (x_3,y_3)

6. **The equations** $x-y+2z=4$, $3x+y+4z=6$, $x+y+z=1$ **have**

1. unique solution 2. infinitely many solutions 3. no solution 4. two solutions

KEY: 2

HINT: $x-y+2z=4, 3x+y+4z=6 \Rightarrow 4x+6z=10$

$3x+y+4z=6, x+y+z=1 \Rightarrow 2x+3z=5$

$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

7. **The locus of the point representing the complex number z for which** $|z+3|^2-|z-3|^2=15$ **is**

1. a circle 2. a parabola 3. a straight line 4. an ellipse

KEY: 3

HINT: $z=x+iy \Rightarrow (x+3)^2+y^2-\{(x-3)^2+y^2\}=15$

$\Rightarrow 12x=15$

8. $\frac{(1+i)^{2016}}{(1-i)^{2014}}=$

1. $-2i$ 2. $2i$ 3. 2 4. -2

KEY: 1

HINT: $\frac{(1+i)^{2016}}{(1-i)^{2014}}=\frac{\{(1+i)^2\}^{1008}}{\{(1-i)^2\}^{1007}}=\frac{(2i)^{1008}}{(-2i)^{1007}}=-2i$

9. **If** $|z_1|=1, |z_2|=2, |z_3|=3$ **and** $|9z_1z_2+4z_1z_3+z_2z_3|=12$, **then the value of** $|z_1+z_2+z_3|$ **is**

1. 3 2. 4 3. 8 4. 2

KEY: 4

HINT: $|z_1|=1, |z_2|=2, |z_3|=3$

$|9z_1z_2+4z_1z_3+z_2z_3|=12$

$z_1\overline{z_1}=(z_1)^2$

$\overline{z_1}=\frac{|z_1|^2}{z_1}$

$\left|(z_1z_2z_3)\left(\frac{9}{z_3}+\frac{4}{z_2}+\frac{1}{z_1}\right)\right|=12$

$(1)(2)(3)\left|\frac{|z_3|^2}{z_3}+\frac{|z_2|^2}{z_2}+\frac{|z_1|^2}{z_1}\right|=12$

$|\overline{z_3}+\overline{z_2}+\overline{z_1}|=2$

$|z_1+z_2+z_3|=2$

10. **If** $1, z_1, z_2, ..., z_{n-1}$ **are the** n^{th} **roots of unity, then** $(1-z_1)(1-z_2)...(1-z_{n-1})=$

1. 0 2. $n-1$ 3. n 4. 1

KEY: 3

HINT: verification (taking n = 3)

$(1-z_1)(1-z_2)...(1-z_{n-1})=n$

11. **If** $12^{4+2x^2} = \left(24\sqrt{3}\right)^{3x^2-2}$**, then** $x =$

1. $\pm\sqrt{\frac{13}{12}}$ 2. $\pm\sqrt{\frac{14}{15}}$ 3. $\pm\sqrt{\frac{12}{13}}$ 4. $\pm\sqrt{\frac{5}{14}}$

KEY: 2

HINT: $12^{4+2x^2} = \left(24\sqrt{3}\right)^{3x^2-2}$

$\left(2\sqrt{3}\right)^{8+4x^2} = \left(2\sqrt{3}\right)^{3\left(3x^2-2\right)}$

$8+4x^2 = 9x^2-6$

$5x^2 = 14$

$x = \pm\sqrt{\frac{14}{5}}$

12. **The product and sum of the roots of the roots of the equation** $\left|x^2\right| - 5|x| - 24 = 0$ **are respectively**

1. -64, 0 2. -24, 5 3. 5, -24 4. 0, 72

KEY: 1

HINT: $\left|x^2\right| - 5|x| - 24 = 0$

$|x|^2 - 8|x| + 3|x| - 24 = 0$

$|x|\left(|x|-8\right) + 3\left(|x|-8\right) = 0$

$|x| = 8, \qquad |x| = -3$

$x = \pm 8 \qquad$ *impossible*

Sum = 0, product = -64

13. **The number of real roots of the equation** $x^5 + 3x^3 + 4x + 30 = 0$ **is**

1. 1 2. 2 3. 3 4. 5

KEY: 1

HINT: $x^5 + 3x^3 + 4x + 30 = 0$

$f(x) + + + +$ no positive root

$f(-x) = -x^5 - 3x^3 - 4x + 30 = 0$ …..+ one-ve root

$\therefore$ number of real roots = 1

14. **If the coefficients of the equation whose roots are k times the roots of the equation** $x^3 + \frac{1}{4}x^2 - \frac{1}{16}x + \frac{1}{144 = 0}$**, are integers then a possible value of k is**

1. 3 2. 12 3. 9 4. 4

KEY: 2

HINT: $f(x) = x^3 + \frac{1}{4}x^2 - \frac{1}{16}x + \frac{1}{144} = 0$

$f\left(\frac{x}{k}\right) = \frac{x^3}{k^3} + \frac{1}{4}\frac{x^2}{k^2} - \frac{1}{16}\frac{x}{k} + \frac{1}{144} = 0$

$= x^3 + \frac{1}{4}kx^2 - \frac{1}{16}k^2x + \frac{k^3}{144}$

K = 4,8,12,16… K = 4,8,12 …

$\therefore$ Possible value of K = 12

15. The sum of all 4-digit numbers that can be formed using the digits 2,3,4,5,6 without repetition, is

1. 533820 2. 532280 3. 533280 4. 532380

KEY: 3

HINT: $\{2,3,4,5,6\}$ sum of four digit number without repetition

$(2+3+4+5+6)\left({}^{5-1}P_{4-1}\right)(1111)(20)(24)(1111)=533280$

16. If a set A has 5 elements, then the number of ways of selecting two subsets P and Q from A such that P and Q are mutually disjoint, is

1. 64 2. 128 3. 243 4. 729

KEY: 3

HINT: $A=\{a_1,a_2,a_3,a_4,a_5\}$ subset P,Q

$\therefore P\cap Q=\phi$ so each element has three possibilities

$\therefore 3\times3\times3\times3\times3=3^5=243$

17. The coefficient of x^4 in the expansion of $\left(1-x+x^2-x^3\right)^4$ is

1. 31 2. 30 3. 25 4. -14

KEY: 1

HINT: $\cot x^4\left(1-x+x^2-x^3\right)^4$

$=\left(1-x+x^2-x^3+x^4-x^5+...x^4+x^5-x^6\right)^4$

$=\left[(1+x)^{-1}-x^4\left(1-x+x^2-x^3+...\right)\right]^4$

$=\left[(1+x)^{-1}-x^4(1+x)^{-1}\right]^4$

$=(1+x)^{-4}\left(1-x^4\right)^4$

$=\left[1-4x+\frac{4(4+1)}{2!}x^2-\frac{4(4+1)(4+2)}{3!}x^3+\frac{4(4+1)(4+2)(4+3)}{4!}x^4.....\right]$

$=x\left(1-4cx^4+4c_2x^8...\right)$

$=-1+35=34$

$\cot x^4=(1)(-4)+\left(\frac{4.5.6.7}{1.2.3.4}\right)(1)$

$=-4+35=31$

18. If the middle term in the expansion of $(1+x)^{2n}$ is the greatest term, then x lies in the interval

1. $\left(\frac{n}{n+1},\frac{n+1}{n}\right)$ 2. $\left(\frac{n+1}{n},\frac{n}{n+1}\right)$ 3. $(n-2,n)$ 4. $(n-1,n)$

KEY: 1

HINT: $(1+x)^{2n}$ middle ten $=T\frac{2n}{2}+1=T_{n+1}$

$P=\frac{Q(2n+1)|X|}{1+|X|}$

$[p]=n$

$n<p<n+1\Rightarrow n<\frac{(2n+1)|x|}{1+|x|}<n+1$

$\frac{n}{2n+1} < \frac{|x|}{1+|x|} < \frac{n+1}{2n+1}$

$\frac{2n+1}{n} > \frac{1+|x|}{|x|} > \frac{2n+1}{n+1}$

$\frac{2n+1}{n} > \frac{1}{|x|} + 1 > \frac{2n+1}{n+1}$

$\frac{2n+1}{n} - 1 > \frac{1}{|x|} > \frac{2n+1}{n+1} - 1 \quad \frac{2x+1-n-1}{n+1}$

$\frac{n+1}{n} > \frac{1}{|x|} > \frac{n}{n+1}$

$\frac{n}{n+1} < |x| < \frac{n+1}{n}$

19. To find the coefficient of x^4 in the expansion of $\frac{3x}{(x-2)(x-1)}$, the interval in which the expansion is valid, is

1. $-2 < x < \infty$ 2. $-\frac{1}{2} < x < \frac{1}{2}$ 3. $-1 < x < 1$ 4. $-\infty < x < \infty$

KEY: 3

HINT: $\frac{3x}{(x-2)(x-1)} = \frac{3x}{+2\left(1-\frac{x}{2}\right)(1-x)}$

$\left|\frac{x}{2}\right| < 1, \qquad |x| < 1$

$|x| < 2, \qquad |x| < 1$

$-2 < x < 2, \qquad -1 < x < 1$

Common interval is $-1 < x < 1$

20. If $(1+\tan\alpha)(1+\tan 4\alpha) = 2,\ \alpha \in \left(0, \frac{\pi}{16}\right)$, then $\alpha =$

1. $\frac{\pi}{20}$ 2. $\frac{\pi}{30}$ 3. $\frac{\pi}{40}$ 4. $\frac{\pi}{60}$

KEY: 1

HINT: $(1+Tan\alpha)(1+Tan4\alpha) = 2, \quad \alpha \in \left(0, \frac{\pi}{16}\right)$

$(1+TanA)(1+TanB) = 2 \Rightarrow A+B = \frac{\pi}{4} \Rightarrow \alpha + 4\alpha = \frac{\pi}{4}$

$\alpha = \frac{\pi}{20}$

21.If $\cos\theta = \frac{\cos\alpha - \cos\beta}{1-\cos\alpha\cos\beta}$, then one of the values of $\tan\left(\frac{\theta}{2}\right)$ is

1) $\cot\frac{\beta}{2}\tan\frac{\alpha}{2}$ 2) $\tan\alpha\tan\frac{\beta}{2}$ 3) $\tan\frac{\beta}{2}\cot\frac{\alpha}{2}$ 4) $\tan^2\frac{\alpha}{2}\tan^2\frac{\beta}{2}$

KEY: 1

HINT:

$$\frac{1}{\cos\theta} = \frac{1-\cos\alpha\cos\beta}{\cos\alpha-\cos\beta}$$

By componendo dividendo

$$\frac{1+\cos\theta}{1-\cos\theta} = \frac{(1+\cos\alpha)}{1-\cos\alpha}\frac{(1-\cos\beta)}{1+\cos\theta}$$

We $\tan^2\frac{\theta}{2} = \tan^2\frac{\alpha}{2}\cot^2\frac{\beta}{2}$.

22. The value of the expression $\frac{1+\sin 2\alpha}{\cos(2\alpha-2\pi)\tan\left(\alpha-\frac{3\pi}{4}\right)} - \frac{1}{4}\sin 2\alpha\left(\cot\frac{\alpha}{2}+\cot\left(\frac{3\pi}{2}+\frac{\alpha}{2}\right)\right)$ **is**

1) 0 2) 1 3) $\sin^2\frac{\alpha}{2}$ 4) $\sin^2\alpha$

KEY: 4

HINT:

$$\frac{1+\frac{2\tan\theta}{1+\tan^2\alpha}}{\frac{1-\tan^2\alpha}{1+\tan^2\alpha}\frac{1+\tan\theta}{1-\tan\alpha}} - \frac{1}{4}\sin 2\alpha.\left[\frac{2}{\tan\alpha}\right]$$

$$\frac{(1+\tan\alpha)^2}{(1+\tan\alpha)^2} - \frac{1}{4}.\frac{4\sin\alpha.\cos\alpha.\cos\alpha}{\sin\alpha}$$

$1 - \cos^2\alpha = \sin^2\alpha$

23. If $\frac{1}{6}\sin\theta, \cos\theta$ **and** $\tan\theta$ **are in geometric progression, then the solution set of** θ **is**

1) $2n\pi \pm \left(\pi/6\right)$ 2) $2n\pi \pm \left(\pi/3\right)$ 3) $n\pi + (-1)^n\left(\pi/3\right)$ 4) $n\pi + \left(\pi/3\right)$

KEY: 2

HINT:

$\cos^2\alpha = \frac{1}{6}\frac{\sin^2\theta}{\cos\theta}$.

$\Rightarrow 6\cos^3\theta = 1 - \cos^2\theta$

$\Rightarrow 6\cos^3\theta + \cos^2\theta - 1 = 0$

$\cos\theta = \frac{1}{2}$ satisfied

$\theta = 2n\pi \pm \frac{\pi}{2}, n \in z$.

24. If $x = \sin(2\tan^{-1}2)$ and $y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$**, then**

1) x > y 2) x = y 3) x = 0 = y 4) x < y

KEY: 1

HINT:

$$x = \sin\left(\sin^{-1}\frac{2(2)}{1+2^2}\right) = \frac{4}{5}$$

Let $\tan^{-1}\frac{4}{3}=\theta$ and $y=\sin\frac{\theta}{2}=\sqrt{\frac{1-\cos\theta}{2}}=\frac{1}{\sqrt{5}}$

$\Rightarrow \quad \tan\theta=\frac{4}{3}$

25. If $\cosh(x)=\frac{5}{4}$, then cosh (3x) =

1) $\frac{61}{16}$ 2) $\frac{63}{16}$ 3) $\frac{65}{16}$ 4) $\frac{61}{63}$

KEY: 3

HINT:

Cosh 3x = 4 $\cosh^3 x$ – 3 cos hx

26. In ΔABC if $x=\tan\left(\frac{B-C}{2}\right)\tan\frac{A}{2}, y=\tan\left(\frac{C-A}{2}\right)\tan\frac{B}{2}$ and $z=\tan\left(\frac{A-B}{2}\right)\tan\frac{C}{2}$ then(x + y + z) =

1) x y z 2) –x y z 3) 2 x y z 4) $\frac{1}{2}xyz$

KEY: 2

HINT:

$x=\frac{b-c}{b+c} \Rightarrow \frac{1+x}{1-x}=\frac{b}{c}$

Similarly $\frac{1+y}{1-y}=\frac{c}{a},\frac{1+z}{1-z}=\frac{a}{b}$

$\therefore \frac{1+x}{1-x}.\frac{1+y}{1-y}.\frac{1+z}{1-z}=1$

$\Rightarrow$ x + y + z = -xyz

27. In ΔABC, if the sides a, b, c are in geometric progression and the largest angle exceeds the smallest angle by 60°, then cosB =

1) $\frac{\sqrt{13}+1}{4}$ 2) $\frac{1-\sqrt{13}}{4}$ 3) 1 4) $\frac{\sqrt{13}-1}{4}$

KEY: 4

HINT:

b^2 = ac $[\therefore A-C=60^\circ]$

$[\therefore A+C=180-B]$

$\Rightarrow \sin^2 B$ = sinA sinC

2 $\sin^2 B$ = 2 sinA sinC

2 – 2 $\cos^2 B$ = cos (A – C) – cos (A + C)

2 – 2 $\cos^2 B$ = cos (60°) + cos B

4 $\cos^2 B$ + 2 cos B – 3 = 0

28. In a ΔABC if ∠A = 90°, then $\cos^{-1}\left(\frac{R}{r_2+r_3}\right)$ is equal to

1) 90° 2) 30° 3) 60° 4) 45°

KEY: 3

HINT: $r_2 + r_3 = 4R\cos^2\frac{A}{2}$ (Put A = 90º)

$$\cos^{-1}\left(\frac{R}{2R}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

29. The Cartesian equation of the plane whose vector equation is

$\bar{\gamma} = (1+\lambda-\mu)\bar{i} + (2-\lambda)\bar{j} + (3-2\lambda+2\mu)\bar{k}$ **where λ, μ are scalars is**

1) 2x + y = 5 2) 2x – y = 5 3) 2x – z = 5 4) 2x + z = 5

KEY: 4

HINT:

$$\bar{r} = [\bar{i} + 2\bar{j} + 3\bar{k}] + \lambda[\bar{i} - \bar{j} - 2\bar{k}] + \mu[-\bar{i} + 2\bar{k}]$$

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & -1 & -2 \\ -1 & 0 & 2 \end{vmatrix} = 0$$

30. For three vectors $\bar{p}, \bar{q}$ and $\bar{r}$, if $\bar{r} = 3\bar{p} + 4\bar{q}$ and $2\bar{r} = \bar{p} - 3\bar{q}$ then

1) $|\bar{r}| < 2|\bar{q}|$ and $\bar{r}, \bar{q}$ have the same direction

2) $|\bar{r}| > 2|\bar{q}|$ and $\bar{r}.\bar{q}$ have opposite directions

3) $|\bar{r}| < 2|\bar{q}|$ and $\bar{r}.\bar{q}$ have opposite direction

4) $|\bar{r}| > 2|\bar{q}|$ and $\bar{r}.\bar{q}$ have the same direction

KEY: 2

HINT: $\bar{r} = 3[2\vec{r} + 3\vec{q}] + 4\vec{q}$

$\Rightarrow -5\vec{r} = 13\vec{q}$

$\Rightarrow \vec{r} = -\frac{13}{5}\vec{q}\,[\vec{r}, \vec{q}$ opposite signs]

$\Rightarrow |\bar{r}| > 2|\bar{q}|$

31. If $\bar{a} = 2\bar{i} + 3\bar{j} - 5\bar{k}, \bar{b} = m\bar{i} + n\bar{j} + 12\bar{k}$ and $\bar{a} \times \bar{b} = 0$, then (m, n) =

1) $\left(\frac{-24}{5}, \frac{-36}{5}\right)$ 2) $\left(\frac{-24}{5}, \frac{36}{5}\right)$ 3) $\left(\frac{24}{5}, \frac{-36}{5}\right)$ 4) $\left(\frac{24}{5}, \frac{36}{5}\right)$

KEY: 1

HINT: $\bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 3 & -5 \\ m & n & 12 \end{vmatrix} = \bar{i}[36+5n] - \bar{j}[24+5m] + \bar{k}[2n-3m] = \bar{0}$

32. If $|\bar{a}| = 3, |\bar{b}| = 4$ and the angle between $\bar{a}$ and $\bar{b}$ is 120º, then $|4\bar{a} + 3\bar{b}|$ is equal to

1) 25 2) 7 3) 13 4) 12

KEY: 4

HINT: $|4\bar{a} + 3\bar{b}|^2 = 16\bar{a}^2 + 9\bar{b}^2 + 24|\bar{a}||\bar{b}|\cos(120^o)$

33. If $\bar{a}, \bar{b}, \bar{c}$ are non-zero vectors such that $(\bar{a} \times \bar{b}) \times \bar{c} = \frac{1}{3}|\bar{b}||\bar{c}|\bar{a}$, $\bar{c} \perp \bar{a}$ and θ is the angle between the vectors $\bar{b}, \bar{c}$ then sin θ=

1) $\frac{2\sqrt{2}}{3}$ 2) $\frac{1}{3}$ 3) $\frac{\sqrt{2}}{3}$ 4) $\frac{2}{3}$

KEY: 1

HINT:

$$(\bar{a}.\bar{c})\bar{b}-(\bar{b}.\bar{c})\bar{a}=\frac{1}{3}|\bar{b}||\bar{c}|\bar{a}$$

$$\Rightarrow 0-(\bar{b}.\bar{c})\bar{a}=\frac{1}{3}|\bar{b}||\bar{c}|\bar{a}$$

$$\Rightarrow -|\bar{b}||\bar{c}|\cos\theta=\frac{1}{3}|\bar{b}||\bar{c}|$$

$$\Rightarrow \cos\theta=-\frac{1}{3}$$

$$\sin\theta=\sqrt{1-\cos^2\theta}=\sqrt{1-\frac{1}{9}}=\sqrt{\frac{8}{9}}=\frac{2\sqrt{2}}{3}.$$

34. If $a(\bar{\alpha}\times\bar{\beta})+b(\bar{\beta}\times\bar{\gamma})+c(\bar{\gamma}\times\bar{\alpha})=\bar{0}$ and atleast one of the scalars a, b, c is non-zero, then the vectors $\bar{\alpha},\bar{\beta},\bar{\gamma}$ are

1) Parallel 2) non coplanar 3) coplanar 4) mutually perpendicular

KEY: 3

HINT:

$\bar{\alpha}\times\bar{\beta},\bar{\beta}\times\bar{\gamma},\bar{\gamma}\times\bar{\alpha}$ ae coplanar

$$[\bar{\alpha}\times\bar{\beta}\ \ \bar{\beta}\times\bar{\gamma}\ \ \bar{\gamma}\times\bar{\alpha}]=0$$

$$\Rightarrow[\bar{\alpha}\ \bar{\beta}\ \bar{\gamma}]^2=0$$

$$\Rightarrow[\bar{\alpha}\ \bar{\beta}\ \bar{\gamma}]=0$$

35. If the mean of 10 observations is 50 and the sum of the squares of the deviations of the observations from the mean is 250, then the coefficient of variation of those observations is

1) 25 2) 50 3) 10 4) 5

KEY: 3

HINT: $\bar{x}=50$

$$\sum(x_i^2-\bar{x})^2=250 \qquad \left[\sigma^2=\frac{1}{10}\times 250=25\right]$$

$$C.V=\frac{S.D}{\bar{x}}\times 100=\frac{5}{50}\times 100=10.$$

36. The variance of the first 50 even natural members is

1) $\frac{833}{4}$ 2) 833 3) 437 4) $\frac{437}{4}$

KEY: 2

HINT:

$$\bar{x}=\frac{2+4+....+100}{50}$$

$$==\frac{50(51)}{50}=51$$

$$\sigma^2=\frac{1}{50}[2^2+4^2+.....+100^2]-(51)^2$$

$$=833$$

37. 3 out of 6 vertices of a regular hexagon are chosen at a time at random. The probability that the triangle formed with these three vertices is an equilateral triangle is

1) $\frac{1}{2}$ 2) $\frac{1}{5}$ 3) $\frac{1}{10}$ 4) $\frac{1}{20}$

KEY: 3

HINT: $n(s) = {}^6C_3$

$n(A) = 2$

$P(A) = \frac{2}{20} = \frac{1}{10}$

38. A speaks truth in 75% of the cases and B in 80% of the cases. Then the probability that their statements about an incident do not match is

1) $\frac{7}{20}$ 2) $\frac{3}{20}$ 3) $\frac{2}{7}$ 4) $\frac{5}{7}$

KEY: 1

HINT: $P(A) = \frac{75}{100}$ $P(B) = \frac{80}{100}$

R.P = $P(A)P(\bar{B}) + P(\bar{A})P(B)$

$= \frac{3}{4}.\frac{1}{5} + \frac{1}{4}.\frac{4}{5} = \frac{7}{20}$

39. If the mean and variance of a binomial distribution are 4 and 2 respectively, then the probability of 2 successes of that binomial variate X, is

1) $\frac{1}{2}$ 2) $\frac{219}{256}$ 3) $\frac{37}{256}$ 4) $\frac{7}{64}$

KEY: 4

HINT: np = 4, npq = 2

then n = 8, p = 1/2, q = 1/2

$p(x = 2) = {}^8C_2\,(1/2)^8 = 7/64$

40. In a city 10 accidents take place in a span of 50 days. Assuming that the number of accidents follow the Poisson distribution, the probability that three or more accidents occur in a day, is

1) $\sum_{k=3}^{\infty}\frac{e^{-\lambda}\lambda^k}{k!}, \lambda = 0.2$ 2) $\sum_{k=3}^{\infty}\frac{e^{\lambda}\lambda^k}{k}, \lambda = 0.2$ 3) $1-\sum_{k=0}^{\infty}\frac{e^{-\lambda}\lambda^k}{k!}, \lambda = 0.2$ 4) $\sum_{k=0}^{3}\frac{e^{-\lambda}\lambda^k}{k!}, \lambda = 0.2$

KEY: 1

HINT: $\lambda = \frac{10}{50} = 0.2$

$P(x \geq 3) = \sum_{k=3}^{\infty}\frac{e^{-\lambda}\lambda^k}{k!}, \lambda = 0.2$

41. Equation of the locus of the centroid of the triangle whose vertices are $(a\cos k, a\sin k)$, $(b\sin k, -b\cos k)$ and (1,0), where k is a parameter, is

1) $(1-3x)^2 + 9y^2 = a^2 + b^2$ 2) $(3x-1)^2 + 9y^2 = 2a^2 + 2b^2$

3) $(3x+1)^2 + (3y)^2 = a^2 + b^2$ 4) $(3x+1)^2 + (3y)^2 = 3a^2 + 3b^2$

KEY:1

Hint : $=(x,y)=\left(\frac{a\cos k+b\sin k+1}{3},\frac{a\sin k-b\cos k+0}{3}\right)$

$\Rightarrow(3x-1)^2+(3y)^2=a^2+b^2$

42. **If the coordinate axes are rotated through an angle $\frac{\pi}{6}$ about the origin, then the transformed equation of $\sqrt{3}x^2-4xy+\sqrt{3}y^2=0$ is**

1) $\sqrt{3}y^2+xy=0$ 2) $x^2-y^2=0$ 3) $\sqrt{3}y^2-xy=0$ 4) $\sqrt{3}y^2-2xy=0$

KEY :3

Hint : $x=x\cos\theta-y\sin\theta, y=x\sin\theta+y\cos\theta$

$x=\frac{\sqrt{3}x-y}{2}, y=\frac{x+\sqrt{3}y}{2}$

43. **If the lines $x+3y-9=0, 4x+by-2=0,$ and $2x-y-4=0$ are concurrent, then the equation of the line passing through the point (b,0) and concurrent with the given lines, is**

1) $2x+y+10=0$ 2) $4x-7y+20=0$ 3) $x-y+5=0$ 4) $x-4y+5=0$

KEY :4

Hint : Point of intersection of $x+3y-9=0$ *and* $2x-y-4=0$ is (3,2) lies on $4x+by-2=0\Rightarrow b=-5$.

$\therefore$ req line is $x-4y+5=0$

44. **The midpoint of the line segment joining the centroid and the orthocentre of the triangle whose vertices are $(a,b),(a,c)$ and (d,c) is**

1) $\left(\frac{5a+d}{6},\frac{b+5c}{6}\right)$ 2) $\left(\frac{a+5d}{6},\frac{5b+c}{6}\right)$ 3) (a,c) 4) $(0,0)$

KEY :1

Hint : $G=\left(\frac{2a+d}{3},\frac{b+2c}{3}\right)$, orthocenter 'O'= (a,c)

$\therefore$ req point = $\frac{O+G}{2}$

45. **The distance from the origin to the image of (1,1) with respect to the line $x+y+5=0$ is**

1) $7\sqrt{2}$ 2) $3\sqrt{2}$ 3) $6\sqrt{2}$ 4) $4\sqrt{2}$

KEY :3

Hint : Image of (1,1) w.r. to x+y+5=0 is P=(-6,-6)

$\therefore$ OP= $6\sqrt{2}$.

46. **The equation of the pair of lines joining the origin to the points of intersection of $x^2+y^2=9$ and $x+y=3$, is**

1) $x^2+(3-y)^2=9$ 2) $(3+y)^2+y^2=9$ 3) $x^2-y^2=9$ 4) $xy=0$

KEY :4

Hint : Homegenising $x^2+y^2=9$ with $x+y=3$

$\Rightarrow x^2+y^2=9\left(\frac{x+y}{3}\right)^2\Rightarrow xy=0$

47. **The orthocenter of the triangle formed by the lines $x+y=1$ and $2y^2-xy-6x^2=0$ is**

1) $\left(\frac{4}{3},\frac{4}{3}\right)$ 2) $\left(\frac{2}{3},\frac{2}{3}\right)$ 3) $\left(\frac{2}{3},\frac{-2}{3}\right)$ 4) $\left(\frac{4}{3},\frac{-4}{3}\right)$

KEY :1

Hint : Lne passing through origin and perpendicular to x+y=1 is x-y=0 (altitude)

Given pair of lines are $(2x-y)(3x+2y)=0$

Point of intersection of $2x-y=0, x+y=1$ is $\left(\frac{1}{3},\frac{2}{3}\right)$.

Another altitude = $y-\frac{2}{3}=\frac{2}{3}\left(x-\frac{1}{3}\right) \Rightarrow 6x-9y+4=0$

$\therefore orthocentre = P.O.I.of\ x-y=0, 6x-9y+4=0 \Rightarrow O=\left(\frac{4}{3},\frac{4}{3}\right)$

48. Let L be the line joining the origin to the point of intersection of the lines represented by $2x^2-3xy-2y^2+10x+5y=0$. If L is perpendicular to the line $kx+y+3=0$, then $k=$

1) $\frac{1}{2}$ 2) $\frac{-1}{2}$ 3) -1 4) $\frac{1}{3}$

KEY :2

Hint : P.O.I. of given pair = (-1,2) = P

Slope of $\overline{OP}=\frac{2-0}{-1-0}=-2$

$m_1 \times m_2 \Rightarrow -2\times -k=-1 \Rightarrow K=\frac{-1}{2}$

49. A circle $S=0$ with radius $\sqrt{2}$ touches the line $x+y-z=0$ at $(1,1.)$ Then the length of the tangent drawn from the point $(1,2.)$ to $S=0$ is

1) 1 2) $\sqrt{2}$ 3) $\sqrt{3}$ 4) 2

KEY :3

Hint : Given question is wrong. Given x+y-z=0
Instead of x+y-2=0

50. The normal drawn at $P(-1,2)$ on the circle $x^2+y^2-2x-2y-3=0$ meets the circle at another point Q. Then the coordinates of Q are

1) $(3,0)$ 2) $(-3,0)$ 3) $(2,0)$ 4) $(-2,0)$

KEY :1

Hint : Centre $=\frac{P+Q}{2} \Rightarrow (1,1)\frac{(-1,2)+Q}{2}$

$Q=(3,0)$ (other end of diameter)

51. If the lines $kx+2y-4=0$ and $5x-2y-4=0$ are conjugate with respect to the circle $x^2+y^2-2x-2y+1=0$, then $k=$

1) 0 2) 1 3) 2 4) 3

KEY :2

Hint : $r^2(l_1l_2+m_1m_2)=(l_1g+m_1f-n_1)(l_2g+m_2f-n_2)$

52. The angle between the tangents drawn from the origin to the circle $x^2+y^2+4x-6y+4=0$ is

1) $\tan^{-1}\left(\frac{5}{13}\right)$ 2) $\tan^{-1}\left(\frac{5}{12}\right)$ 3) $\tan^{-1}\left(\frac{12}{5}\right)$ 4) $\tan^{-1}\left(\frac{13}{5}\right)$

KEY :3

Hint : $Tan\left(\frac{\theta}{2}\right)=\frac{r}{\sqrt{S_{11}}}$

53. **If the angle between the circles** $x^2+y^2-2x-4y+c=0$ **and** $x^2+y^2-4x-2y+4=0$ **is** 60^o**, then** c **is equal to**

1) $\frac{3\pm\sqrt{5}}{2}$ 2) $\frac{6\pm\sqrt{5}}{2}$ 3) $\frac{9\pm\sqrt{5}}{2}$ 4) $\frac{7\pm\sqrt{5}}{2}$

KEY :4

Hint : $\cos\theta=\frac{d^2-r_1^2-r_2^2}{2r_1r_2}$

54. **A circle** S **cuts three circles** $x^2+y^2-4x-2y+4=0$, $x^2+y^2-2x-4y+1=0$, **and** $x^2+y^2+4x+2y+1=0$ **orthogonally. Then the radius of** S **is**

1) $\sqrt{\frac{29}{8}}$ 2) $\sqrt{\frac{28}{11}}$ 3) $\sqrt{\frac{29}{7}}$ 4) $\sqrt{\frac{29}{5}}$

KEY :1

Hint : Find the Radical centre and then find $\sqrt{S_{11}}$.

55. **The distance between the vertex and the focus of the parabola** $x^2-2x+3y-2=0$ **is**

1) $\frac{4}{5}$ 2) $\frac{3}{4}$ 3) $\frac{1}{2}$ 4) $\frac{5}{6}$

KEY :2

Hint : $(x-1)^2=-3(y-1)\Rightarrow 4a=-3\Rightarrow a=-3/4$
$SA=|a|=3/4$

56. **If** (x_1,y_1) **and** (x_2,y_2) **are the end points of a focal chord of the parabola** $y^2=5x$**, then** $4x_1x_2+y_1y_2=$

1) 25 2) 5 3) 0 4) $\frac{5}{4}$

KEY :3

Hint : $t_1t_2=-1\Rightarrow x_1x_2a^2, y_1y_2=-4a^2$
$\therefore 4x_1x_2+y_1y_2=0$

57. **The distance between the focii of the ellipse** $x=3\cos\theta, y=4\sin\theta$ **is**

1) $2\sqrt{7}$ 2) $7\sqrt{2}$ 3) $\sqrt{7}$ 4) $3\sqrt{7}$

KEY :1

Hint : $\frac{x^2}{9}+\frac{y^2}{16}=1\Rightarrow SS^1=2bc=2\sqrt{7}$ $\frac{x^2}{9}+\frac{y^2}{16}=1$

58. **The equation of the latus recta of the ellipse** $9x^2+25y^2-36x+50y-164=0$ **are**

1) $x-4=0, x+2=0$ 2) $x-6=0, x+2=0$ 3) $x+6=0, x-2=0$ 4) $x+4=0, x+5=0$

KEY :2

Hint : Given ellipse is $\frac{(x-2)^2}{25}+\frac{(y+1)^2}{9}=1$
$x=h\pm ae\Rightarrow x-6=0, x+2=0$ $x=h\pm ae$

59. **The values of the m for which the line $y = mx + 2$ becomes a tangent to the hyperbola $4x^2 - 9y^2 = 36$ is**

1) $\pm\frac{2}{3}$ 2) $\pm\frac{2\sqrt{2}}{3}$ 3) $\pm\frac{8}{9}$ 4) $\pm\frac{4\sqrt{2}}{3}$

KEY :2

Hint : $c^2 = a^2m^2 - 6^2 \Rightarrow m = \pm\frac{2\sqrt{2}}{3}$.

60. **The harmonic conjugate of $(2,3,4)$ with respect to the points $(3,-2,2),(6,-17,-4)$ is**

1) $\left(\frac{1}{2},\frac{1}{3},\frac{1}{4}\right)$ 2) $\left(\frac{18}{5},-5,\frac{4}{5}\right)$ 3) $\left(\frac{-18}{5},\frac{5}{4},\frac{4}{5}\right)$ 4) $\left(\frac{18}{5},-5,\frac{-4}{5}\right)$

KEY :2

Hint : $P = (2,3,4), A = (3,-2,2), B = (6,-17,-4)$

'P' divides $\overline{AB}$ in the ratio $= 1:-4 \Rightarrow$ 'Q' divides $\overline{AB}$ in the ratio 1:4 $Q = \frac{B+4A}{5}$

61. **If a line makes angles α, β, γ and δ with the four diagonals of a cube, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta$ is**

1) $\frac{4}{3}$ 2) $\frac{8}{3}$ 3) $\frac{7}{3}$ 4) $\frac{5}{3}$

KEY: 2

HINT: $w.k.t\ \cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$

$\therefore \sin^2\alpha + \sin^2\beta + \sin^2\gamma + \sin^2\delta = 4 - \frac{4}{3} = \frac{8}{3}$

62. **If the plane $56x + 4y + 9z = 2016$ meets the coordinate axes in A, B, C then the centroid of the triangle ABC is**

1) $(12,168,224)$ 2) $(12,168,112)$

3) $\left(12,168,\frac{224}{3}\right)$ 4) $\left(12,-168,\frac{224}{3}\right)$

KEY: 3

HINT : Given plane **is** $56x + 4y + 9z = 2016$

Convert to $\Rightarrow \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

Then A, B, C. $\Rightarrow G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}, \frac{z_1 + z_2 + z_3}{3}\right)$

63. **The value(s) of x for which the function** $f(x) = \begin{cases} 1-x & , \quad x < 1 \\ (1-x)(2-x) & , \quad 1 \le x \le 2 \\ 3-x & , \quad x > 2 \end{cases}$ **fails to be continuous is (are)**

1) 1 2) 2 3) 3 4) All real numbers

KEY: 2

HINT : **Discontinuous at** $x = 2$

$R.H.L = \underset{x\to 2^+}{Lt} f(x) = 1$ $L.H.L = \underset{x\to 2^-}{Lt} f(x) = 0$

$L.H.L \ne R.H.L$

64. $\underset{x \to 0}{Lt} \frac{6^x - 3^x - 2^x + 1}{x^2} =$

1) $(\log_e 2)\log_e 3$ 2) $\log_e 5$

3) $\log_e 6$ 4) 0

KEY: 1

HINT : $\underset{x \to 0}{Lt} \frac{3^x(2^x - 1) - 1(2^x - 1)}{x^2} = \underset{x \to 0}{Lt} \frac{(2^x - 1)}{x}\frac{(3^x - 1)}{x} = \log 2.\log 3$

65. **Define** $f(x) = \begin{cases} x^2 + bx + c & , x < 1 \\ x & , x \ge 1 \end{cases}$. **If** $f(x)$ **is differentiable at** $x = 1$, **then** $(b - c) =$

1) -2 2)0 3) 1 4) 2

KEY:1

HINT: R.H.D = 1, **L.H.D =** $\underset{x \to 1^-}{Lt} \frac{(x^2 + bx + c) - 1}{x - 1} = 1$

$= \underset{x \to 1-}{Lt}\left(\frac{(x-1)(x+1)}{x-1} + \frac{bx}{x-1} + \frac{c}{x-1}\right) = 1$

$2 + \underset{x \to 1^-}{Lt} \frac{b}{1} + 0 = 1$

$b = -1$

$b - c = -1 - 1 = -2$

66. **If** $x = a$ **is a root of multiplicity two of a polynomial equation** $f(x) = 0$**, then**

1) $f'(a) = f''(a) = 0$ 2) $f''(a) = f(a) = 0$

3) $f'(a) \ne 0 \ne f''(a)$ 4) $f(a) = f'(a) = 0; f''(a) \ne 0$

KEY: 4

HINT: Conceptual

67. **If** $y = \log_2(\log_2 x)$, **then** $\frac{dy}{dx} =$

1) $\frac{\log_e 2}{x \log_e x}$ 2) $\frac{1}{\log_e (2x)^x}$

3) $\frac{1}{(x \log_e x)\log_e 2}$ 4) $\frac{1}{x(\log_2 x)^2}$

KEY: 3

HINT: $2^y = \frac{\log x}{\log 2}$ and then differentiate

68. **The angle of intersection between the curves** $y^2 + x^2 = a^2\sqrt{2}$ **and** $x^2 - y^2 = a^2$**, is**

1) $\frac{\pi}{3}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{6}$ 4) $\frac{\pi}{12}$

KEY: 2

HINT: $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$

69. **If** $A > 0, B > 0$ **and** $A + B = \frac{\pi}{3}$, **then the maximum value of** $\tan A \tan B$ **is**

1) $\frac{1}{\sqrt{3}}$ 2) $\frac{1}{3}$ 3) $\frac{1}{2}$ 4) $\sqrt{3}$

KEY: 2

HINT: $A+B=\frac{\pi}{3}$

Put $A=B=\frac{\pi}{6}$

$\tan A.\tan B=\frac{1}{\sqrt{3}}.\frac{1}{\sqrt{3}}=\frac{1}{3}$

70. The equation of the common tangent drawn to the curves $y^2=8x$ and $xy=-1$ is

1) $y=2x+1$ 2) $2y=x+6$ 3) $y=x+2$ 4) $3y=8x+2$

KEY: 3

HINT: $y^2=8x,\ y=mx+\frac{a}{m}$

$y=mx+\frac{2}{m}$. It also touches $xy=-1$. Then $m=1$

$\therefore\ y=x+2$

71. Suppose $f(x)=x(x+3), x\in[-1,4]$, Then a value of c in $(-1,4)$ satisfying $f'(c)=10$ is

1) 2 2) $\frac{5}{2}$ 3) 3 4) $\frac{7}{2}$

KEY: 1

HINT: $f(x)=x\left(x^2+x-6\right)=x^3+x^2-6x$

$f'(x)=3x^2+2x^2-6=10$

$\Rightarrow 3x^2+2x-16=0$

$x(3x+8)-2(3x+8)=0$

$(3x+8)(x-2)=0$

$x=2\ or\ \frac{-8}{3}$

72. If $\int x^3e^{5x}dx=\frac{e^{5x}}{5^4}(f(x))+c$ then $f(x)=$

1) $\frac{x^3}{5}-\frac{3x^2}{5^2}+\frac{6x}{5^3}-\frac{6}{5^4}$ 2) $5x^3-5^2x^2+5^3x-6$

3) $5^2x^3-15x^2+30x-6$ 4) $5^3x^3-75x^2+30x-6$

KEY: 4

HINT: Integration by parts

73. $\int\frac{x}{\left(x^2+2x+2\right)^2}dx=$

1) $\frac{x^2+2}{x^2+2x+2}-\frac{1}{2}\tan^{-1}(x+1)+c$ 2) $\frac{x^2+2}{2\left(x^2+2x+2\right)}-\frac{1}{2}\tan^{-1}(x-1)+c$

3) $\frac{x^2-2}{4\left(x^2+2x+2\right)}-\frac{1}{2}\tan^{-1}(x+1)+c$ 4) $\frac{2(x-1)}{\left(x^2+2x+2\right)}+\frac{1}{2}\tan^{-1}(x+1)+c$

KEY: 1

HINT: $x+1=\tan\theta$

74. **If** $\int \log\left(a^2+x^2\right)dx = h(x)+c$, **then** $h(x)$ =

1) $x\log\left(x^2+x^2\right)+2\tan^{-1}\left(\frac{x}{a}\right)$

2) $x^2\left(x^2+x^2\right)+2\tan^{-1}\left(\frac{x}{a}\right)$

3) $x\log\left(a^2+x^2\right)-2a\tan^{-1}\left(\frac{x}{a}\right)$

4) $x^2\left(a^2+x^2\right)+2x-a^2\tan^{-1}\left(\frac{x}{a}\right)$

KEY: 3

HINT: Integration by parts

75. **For** $x>0$, **if** $\int(\log x)^5\,dx = x\left[A(\log x)^5+B(\log x)^4+C(\log x)^3+D(\log x)^2+E(\log x)+F\right]+$ **constant then** $A+B+C+D+E+F=$

1) -44 2) -42 3) -40 4) -36

KEY: 1

HINT: Reduction formulae

76. **The area included between the parabola** $y=\frac{x^2}{4a}$ **and the curve** $y=\frac{8a^3}{\left(x^2+4a^2\right)}$ **is**

1) $a^2\left(2\pi+\frac{2}{3}\right)$ 2) $a^2\left(2\pi-\frac{8}{3}\right)$ 3) $a^2\left(\pi+\frac{4}{3}\right)$ 4) $a^2\left(2\pi-\frac{4}{3}\right)$

KEY: 4

HINT: P.I. of the x-coordinates -2a and 2a

Required area $A=\int_{-2a}^{2a}\left(\frac{8a^3}{x^2+4a^2}-\frac{x^2}{4a}\right)dx$

$=2\int_0^{2a}\left(\frac{8a^3}{x^2+4a^2}-\frac{x^2}{4a}\right)dx$

$=a^2\left(2\pi-\frac{4}{3}\right)$

77. **By the definition of the definite integral, the value of**

$\lim_{n\to\infty}\left(\frac{1}{\sqrt{n^2-1}}+\frac{1}{\sqrt{n^2-2^2}}+.....+\frac{1}{\sqrt{n^2-(n-1)^2}}\right)$ **is equal to**

1) π 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{6}$

KEY: 2

HINT: $\underset{x\to\infty}{Lt}\sum_{r=1}^{(n-1)}\frac{1}{\sqrt{n^2-r^2}}$

$= \underset{x\to\infty}{Lt} \sum_{r=1}^{(n-1)} \frac{1}{n} \frac{1}{\sqrt{1-\left(\frac{r}{n}\right)^2}} = \int_0^1 \frac{1}{\sqrt{1-x^2}} dx = \left(\sin^{-1} x\right)_0^1 = \frac{\pi}{2}$

78. $\int_{-\pi/4}^{\pi/4} \left(\frac{x+\frac{\pi}{4}}{2-\cos 2x} \right) dx =$

1) $\frac{8\pi\sqrt{3}}{5}$ 2) $\frac{2\pi\sqrt{3}}{9}$ 3) $\frac{4\pi^2\sqrt{3}}{9}$ 4) $\frac{\pi^2}{6\sqrt{3}}$

KEY: 4

HINT: $\int_{-\pi/4}^{\pi/4} \frac{x}{2-\cos 2x} dx + \frac{\pi}{4} \int_{-\pi/4}^{\pi/4} \frac{1}{2-\cos 2x} dx$

$= 0 + \frac{\pi}{2} \int_0^{\pi/4} \frac{1}{2-\cos 2x} dx$ put $\tan x = t$ and then integrate

$= \frac{\pi^2}{6\sqrt{3}}$ $\left(x - e^{\tan^{-1} y}\right) \frac{dy}{dx} = -\left(1+y^2\right)$

79. The solution of the differential equation $\left(1+y^2\right) + \left(x - e^{\tan^{-1} y}\right) \frac{dy}{dx} = 0,$ **is**

1) $x e^{\tan^{-1} y} = \tan^{-1} y + c$ 2) $x e^{2\tan^{-1} y} = e^{-\tan^{-1} y} + c$

3) $2x e^{\tan^{-1} y} = e^{2\tan^{-1} y} + c$ 4) $x^2 e^{\tan^{-1} y} = 4e^{2\tan^{-1} y} + c$

KEY: 3

HINT: $\left(x - e^{\tan^{-1} y}\right) \frac{dy}{dx} = -\left(1+y^2\right)$

$-\left(\frac{x - e^{\tan^{-1} y}}{1+y^2} \right) = \frac{dx}{dy}$

$\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{\tan^{-1} y}}{1+y^2}$

I.F. $= e^{\tan^{-1} y}$

G.S = $x.e^{\tan^{-1} y} = \int \frac{e^{\tan^{-1} y}}{1+y^2} . e^{\tan^{-1} y} \, dy$ put $\tan^{-1} y = t$ and then integrate.

$\Rightarrow 2x.e^{\tan^{-1} y} = e^{2\tan^{-1} y} + c$

80. The solution of the differential equation $(2x-4y+3)\frac{dy}{dx} + (x-2y+1) = 0$ **is**

1) $\log((2x-4y)+3) = x-2y+c$ 2) $\log[2(2x-4y)+3] = 2(x-2y)+c$

3) $\log[2(x-2y)+5] = 2(x+y)+c$ 4) $\log[4(x-2y)+5] = 4(x+2y)+c$

KEY: 4

HINT: $\frac{dy}{dx} = \frac{-[(x-2y)+1]}{2(x-2y)+3}$ put $x-2y=t \Rightarrow \frac{dy}{dx} = \frac{1}{2}\left(1 + \frac{dt}{dx}\right)$

$$\Rightarrow \frac{1}{2}\left(1+\frac{dt}{dx}\right)=\frac{-(t+1)}{2t+3}$$

$$\Rightarrow \int \frac{2t+3}{4t+5}dt=\int dx$$

$$\int\left[\frac{1}{2}+\frac{\frac{1}{2}}{4t+5}\right]dt=\int dx$$

And then integrate and substitute $t=x-2y$

$$\Rightarrow \log\left[4(x-2y)+5\right]=4(x+2y)+c$$

PHYSICS

81. Match the list-I with list-II

List-I	**List-II**
A) Boltzmann constant	**I)** $ML^{\circ}T^{\circ}$
B) Coefficient of viscosity	**II)** $ML^{-1}T^{-1}$
C) Water equivalent	**III)** $MLT^{-3}K^{-1}$
D) Coefficient of thermal conductivity	**IV)** $ML^{2}T^{-2}K^{-1}$

(1) A-III; B-I; C-II; D-IV **(2) A-III; B-II; C-I; D-IV**
(3) A-IV; B-II; C-I; D-III **(4) A-IV; B-I;C-II; D-III**

KEY: 3

HINT: Boltzmann's constant

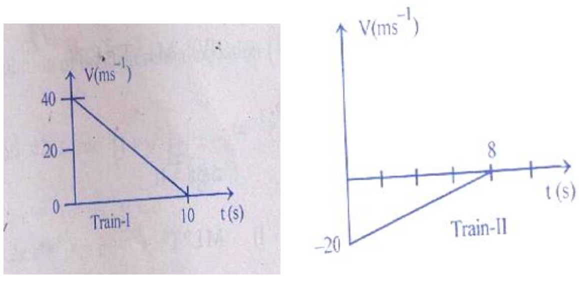
$$K=\frac{2E}{3T}$$

$$K=\frac{ML^2T^{-2}}{K}=ML^2T^{-2}K^{-1}$$

Coefficient of viscosity $\eta=\dfrac{F}{A\frac{dv}{dx}}$

$$\eta=\frac{F}{A\frac{dv}{dx}}=\frac{MLT^{-2}}{L^2T^{-1}}=ML^{-1}T^{-1}$$

82. Two trains, which are moving along different tracks in opposite directions are put on the same track by mistake. On noticing the mistake, when the trains are 300 m apart the drivers start slowing down the trains. The graphs given below show decrease in their velocities as function of time. The separation between the trains when both have stopped is



(1) 120m **(2) 20m** **(3) 60m** **(4) 280m**

KEY: 2

HINT: From given graph retardation of two trains are

$a_1 = 4ms^{-2}$, $a_2 = 2.5ms^{-2}$ (Slope of given graphs)

Stopping distances

$$x_1 = \frac{40\times 40}{2\times 4} = 200m$$

$$x_2 = \frac{20\times 20}{2\times 2.5} = 80m$$

$$d = D-(x_1+x_2)$$

$$= 300-(200+80) = 20m$$

83. A point object moves along an arc of a circle of radius 'R'. Its velocity depends upon the distance covered 'S' as $V = K\sqrt{S}$ where 'K' is a constant. If $'\theta'$ is the angle between the total acceleration and tangential acceleration, then

(1) $\tan\theta = \sqrt{\frac{S}{R}}$ **(2)** $\tan\theta = \sqrt{\frac{S}{2R}}$ **(3)** $\tan\theta = \frac{S}{2R}$ **(4)** $\tan\theta = \frac{2S}{R}$

KEY: 4

HINT:

$$V = K\sqrt{S}$$

$$a_r = \frac{v^2}{R} = \frac{K^2S}{R}$$

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(K\sqrt{S}) = \frac{K^2}{2}$$

$$\tan\theta = \frac{a_r}{a_t} = \frac{2S}{R}$$

84. A body projected from the ground reaches a point 'X' in its path after 3 seconds and from there it reaches the ground after further 6 seconds. The vertical distance of the point 'X' from the ground is

(acceleration due to gravity $=10ms^{-2}$)

(1) 30m (2) 60m (3) 80m (4) 90m

KEY: 4

HINT:

Let u be velocity of projection

Time of flight $T = \frac{2u}{g} = 9\sec$

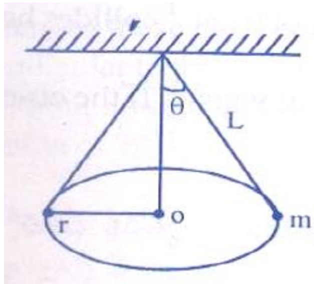
$$u = \frac{9\times 10}{2} = 45m/s$$

$$x = ut - \frac{1}{2}gt^2$$

$$= 45(3) - \frac{1}{2}(10)9$$

$$= 135-45 = 90m$$

85. A particle of mass 'm' is suspended from a ceiling through a string of length 'L'. If the particle moves in a horizontal circle of radius 'r' as shown in the figure, then the speed of the particle is



(1) $r\sqrt{\frac{g}{\sqrt{L^2-r^2}}}$ **(2)** $g\sqrt{\frac{r}{\sqrt{L^2-r^2}}}$ **(3)** $r\sqrt{\frac{g}{L^2-r^2}}$ **(4)** $g\sqrt{\frac{r}{L^2-r^2}}$

KEY: 1

HINT:

According to dimensional analysis first option is correct

$$r\sqrt{\frac{g}{\sqrt{L^2-r^2}}} = L\sqrt{\frac{LT^{-2}}{L}} = LT^{-1} = \text{velocity}$$

86. A particle is placed at rest inside a hollow hemisphere of radius 'R'. The co-efficient of friction between the particle and the hemisphere is $\mu = \frac{1}{\sqrt{3}}$. The maximum height upto which the particle can remain stationary is

(1) $\frac{R}{2}$ **(2)** $\left(1-\frac{\sqrt{3}}{2}\right)R$ **(3)** $\frac{\sqrt{3}}{2}R$ **(4)** $\frac{3R}{8}$

KEY: 2

HINT:

$$h = R\left[1-\frac{1}{\sqrt{\mu^2+1}}\right]$$

$$= R\left[1-\frac{1}{\sqrt{\frac{1}{3}+1}}\right]$$

$$= R\left[1-\frac{\sqrt{3}}{2}\right]$$

87. A 1kg ball moving with a speed of $6ms^{-1}$ collides head-on with a 0.5 kg ball moving in the opposite direction with a speed of $9ms^{-1}$. If the co-efficient of restitution is $\frac{1}{3}$, the energy lost in the collision is

(1) $303.4J$ **(2)** $66.7J$ **(3)** $33.3J$ **(4)** $67.8J$

KEY: 3

HINT:

$m_1 = 1kg$

$m_2 = 0.5kg$

$u_1 = 6m/s$

$u_2 = 9m/s$

$U = \frac{1}{2}\frac{m_1 m_2}{m_1 + m_2}(u_1 + u_2)^2(1 - e^2)$

$= 33.3J$

88. A ball is thrown vertically down from a height of 40 m from the ground with an initial velocity 'v'. The ball hits the ground, loses $\frac{1^{rd}}{3}$ of its total mechanical energy and rebounds back to the same height. If the acceleration due to gravity is $10ms^{-2}$, the value of 'v' is

(1) $5ms^{-1}$ **(2)** $10ms^{-1}$ **(3)** $15ms^{-1}$ **(4)** $20ms^{-1}$

KEY: 4

HINT: $KE_i = mgh + \frac{1}{2}mv^2$

$KE_i = \left(400 + \frac{1}{2}v^2\right)m$

$KE_f = \frac{2}{3}\left[400m + \frac{1}{2}mv^2\right] = m \times 10 \times 40$

$\frac{2}{3}\left[400 + \frac{v^2}{2}\right] = 400$

$400 + \frac{v^2}{2} = 600$

$\frac{v^2}{2} = 200$

$v = 20m/s$

89. Three identical uniform thin metal rods from the three sides of an equilateral triangle. If the moment of inertia of the system of these three rods about an axis passing through the centroid of the triangle and perpendicular to the plane of the triangle is 'n' times the moment of inertia of one rod separately about an axis passing through the centre of the rod and perpendicular to its length, the value of 'n' is

(1) 3 (2) 6 (3) 9 (4) 12

KEY: 2

HINT:

$\tan 30° = \frac{x}{\frac{a}{2}}$

$x = \frac{a}{2\sqrt{3}} = \frac{l}{2\sqrt{3}}$

$I = \frac{ml^2}{12} + \frac{ml^2}{4 \times 3} = \frac{ml^2}{6}$

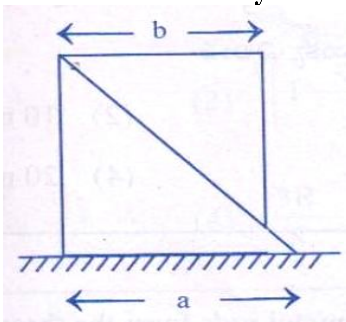
$I_{net} = \frac{3ml^2}{6} = \frac{ml^2}{2}$

$$I_{net} = n\frac{ml^2}{12}$$

$$\frac{ml^2}{2} = n\frac{ml^2}{12}$$

$$n = 6$$

90. Two smooth and similar right angled prisms are arranged on a smooth horizontal plane as shown in the figure. The lower prism has a mass '3' times the upper prism. The prisms are held in an initial position as shown and are then released. As the upper prism touches the horizontal plane, the distance moved by the lower prism is



(1) $a-b$ **(2)** $\frac{a-b}{3}$ **(3)** $\frac{b-a}{2}$ **(4)** $\frac{a-b}{4}$

KEY: 4

HINT:

$$x = \frac{ml\cos\theta}{m_1+m_2}$$

$$x = \frac{ml\cos\theta}{m_1+m_2} = \frac{ml\cos\theta}{m+3m}$$

Relative displacement $= \frac{a-b}{4}$

91. A particle is executing simple harmonic motion with an amplitude of 2m. The difference in the magnitudes of its maximum acceleration and maximum velocity is 4. The time period of its oscillation and its velocity when it is 1m away from the mean position are respectively.

(1) $2s, 2\sqrt{3}ms^{-1}$ (2) $\frac{7}{22}s, 4\sqrt{3}ms^{-1}$ (3) $\frac{22}{7}s, 2\sqrt{3}ms^{-1}$ (4) $\frac{44}{7}s, 4\sqrt{3}ms^{-1}$

KEY: 3

SOL: $Aw^2 - Aw = 4$

$$w^2 - w = \frac{4}{2} = 2$$

$$w[w-1] = 2, \Rightarrow w = 2$$

$$V = w\sqrt{A^2 - \frac{A^2}{4}}$$

$$V = w\sqrt{\frac{3A^2}{4}} = w\sqrt{3} \Rightarrow V = 2\sqrt{3} \Rightarrow T = \frac{2\pi}{w} = \frac{2\pi}{2} = \pi$$

92. **Two bodies of masses 'm' and '9m' are placed at a distance 'r'. The gravitational potential at a point on the line joining then, where gravitational field is zero, is(G is universal gravitational constant)**

(1) $\frac{-14GM}{r}$ (2) $\frac{-16GM}{r}$ (3) $\frac{-12GM}{r}$ (4) $\frac{-8GM}{r}$

KEY: 2

SOL: $x = \frac{r}{\sqrt{\frac{m_2}{m_1}}+1}$

$V = \frac{-Gm}{r}.4 - \frac{G.9m}{3r} \text{x} 4$

$V - \frac{4Gm}{r} - \frac{12Gm}{r}$, $-16\frac{Gm}{r}$

93. **When a load of 80N is suspended from a string, its length is 101mm. If a load of 100N is suspended, its length is 102mm. If a load of 160N is suspended from it, then length of the string is (Assume the area of cross-section unchanged)**

(1) 15.5 cm (2) 13.5 cm (3) 16.5 cm (4) 10.5 cm

KEY: 4

SOL: $80 \propto 101 - l$

$100 \propto 102 - l$

$\frac{4}{5} - \frac{101-l}{102-l}$

$505 - 5l = 408 - 4l$

$505 - 408 = l$

$l = 97m$

Now $160 \propto x - 97$

$\frac{100}{160} = \frac{5}{x-97}$

$8 = x - 97$

$x = 105mm$

$x = 10.5cm$

94. **A sphere of material of relative density 8 has a concentric spherical cavity and just sinks in water. If the radius of the sphere is 2cm, then the volume of the cavity is**

(1) $\frac{76}{3}cm^3$ (2) $\frac{79}{3}cm^3$ (3) $\frac{82}{3}cm^3$ (4) $\frac{88}{3}cm^3$

KEY: 4

SOL: $\frac{4}{3}\pi r^3 \text{ x } 1 \text{ x } g = V \text{ x } 8 \text{ x } g$

$\frac{1}{6} \text{ x } \pi \text{ x } 2^3 = V \Rightarrow \quad \frac{4\pi}{3} = V$

$V_c = \frac{4}{3}\pi r^3 - \frac{4\pi}{3} \Rightarrow \frac{4}{3}\pi\left[2^3 - 1\right]$

$\Rightarrow \frac{4}{3} \text{x} \frac{22}{7} \text{x} 7 \Rightarrow \quad \frac{88}{3}cm^3$

95. **A hunter fired a metallic bullet of mass 'm' kg from a gun towards an obstacle and it just melts when it is stopped by the obstacle. The initial temperature of the bullet is 300K. If $\frac{1}{4}$th of heat is absorbed by the obstacle, then the minimum velocity of the bullet is**
[Melting point of bullet = 600K,
Specific heat of bullet = $0.03\,cal\,g^{-1}\,^{o}C^{-1}$,
Latent heat of fusion of bullet = $6\,cal\,g^{-1}$]

(1) $410ms^{-1}$ (2) $260ms^{-1}$ (3) $460ms^{-1}$ (4) $310ms^{-1}$

KEY: 1

SOL: $\frac{3}{4}(K.E) = ms\Delta\theta + mL$

$\frac{3}{4}\left(\frac{1}{2}mv^2\right) = ms[600-300] + mL$

$\frac{3}{8}\,x\,V^2 = 0.03\,x\,4200\,x\,300 + 6\,x\,4200$

$\Rightarrow 900\,x\,42 + 25200$

$V^2 = 63000\,x\,\frac{8}{3}$

$V^2 = 168000$

$V = 409.87m/s$

410ml

96. **'M' kg of water at 't' ^{o}C is divided into two parts so that one part of mass 'm' kg when converted into ice at 0^0C would release enough heat to vapourise the other part, then $\frac{m}{M}$ is equal to**
[Specific heat of water = $1cal\,g^{-1}\,^{o}C^{-1}$,
Latent heat of fusion of ice = $1cal\,g^{-1}\,^{o}C^{-1}$,
Latent heat of steam = $540\,cal\,g^{-1}$]

(1) 640 – t (2) $\frac{720-t}{640}$ (3) $\frac{640+t}{720}$ (4) $\frac{640-t}{720}$

KEY: 4

SOL: $mc_wt + mL_{ice} = (M-m)C_W(100-t) + (M-m)L_{steam}$

$\therefore \frac{m}{M} = \frac{640-t}{720}$

97. **A diatomic gas $(\gamma = 1.4)$ does 300J work when it is expanded isobarically. The heat given to the gas in this process is**

(1) 1050 J (2) 950 J (3) 600 J (4) 550 J

KEY: 1

SOL: $(dq) = nc_vdT + dw$

$P = n\frac{R}{rT}dT + dw$

$\frac{300}{\left(\frac{7}{5}-1\right)} + 300 \Rightarrow (dq)_p = \frac{300}{0.4} + 300 = 1050\,joule$

98. When the absolute temperature of the source of a Carnot heat engine is increased by 25% its efficiency increases 80%. The new efficiency of the engine is

(1) 12% (2) 24% (3) 48% (4) 36%

KEY: 4

SOL: $n = \frac{T_1 - T_2}{T_1}$

$100 = \frac{100 - T_2}{100}$

$T_2 = 80k$

$180 = \frac{125 - T_2}{125}$

$n_2 = \frac{T_1 - T_2}{T_1} \text{x} 100$

$= \frac{125 - 80}{125} = \frac{45}{125} = \frac{9}{25} \text{x} 100$

36%

99. A cylinder of fixed capacity 67.2 litres contains helium gas at STP. The amount of heat needed to rise the temperature of the gas in the cylinder by 20^0 C is $\left(R = 8.31 J\, mol^{-1} K^{-1}\right)$

(1) 748 J (2) 374 J (3) 1000 J (4) 500 J

KEY: 1

SOL: $(dq)_v = nc_v dT$

$= 3 \text{x} \frac{3R}{2} \text{x} 20$

=90R

90 x 8.3

748joule

100. For a certain organ pipe, three successive resonance frequencies are observed at 425, 595 and 765Hz, respectively. The length of the pipe is (speed of sound in air =340 ms^{-1})

(1) 0.5 m (2) 1 m (3) 1.5 m (4) 2 m

KEY: 4

SOL: $(2n+1)\frac{V}{4l} = 425$

$\left[2(n+1)+1\right]\frac{V}{4l} = 595$

$(2n+3)\frac{V}{4l} = 595$

$2\frac{V}{4l} = 170$

101. A student holds a tuning fork oscillating at 170 Hz. He walks towards a wall at a constant speed of 2*ms*$^{-1}$. The beat frequency observed by the student between the tuning fork and its echo is (Velocity of sound == $342 ms^{-1}$)

1) 2.5Hz 2) 3Hz 3)1Hz 4) 2Hz

KEY : 4

Hint : $\Delta n = \left(\frac{2u}{v-u}\right)_n$

102. **An infinitely long rod lies along the axis of a concave mirror of focal length 'f'. the nearer end of the rod is at a distance u, $(u > f)$ from the mirror. It's image will have a length**

1) $\frac{uf}{u+f}$ 2) $\frac{uf}{u-f}$ 3) $\frac{f^2}{u+f}$ 4) $\frac{f^2}{u-f}$

KEY : 4

Hint : Volume – 1

103. **In Yooung's double slit experiment, red light of wavelength 6000A is used and then n^th bright fringe is obtained at a point 'P' on the screen. Keeping the same setting, the source of light is replaced by green light of wavelength 5000A and now $(n+1)^{th}$ bright fringe is obtained at the point P on the screen. The value of 'n' is**

1) 4 2) 5 3) 6 4) 3

KEY : 2

Hint : $n_1\lambda_1 = n_2\lambda_2$

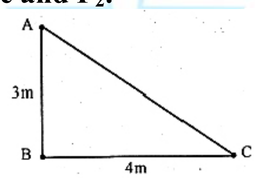
104. **Two charges each of charge +10c are kept on Y-axis at *y*=-*a* and *y*=+*a* respectively. Another point charge □20 *c* is placed at the origin and given a small displacement $x(x << a)$ along X-axis. The force acting on the point charge is (x and a are in metres, $\frac{1}{4\pi\in_0} = 9\times10^9 Nm^2C^{-2}$)**

1) $\frac{3.6x}{a^2}N$ 2) $\frac{2.4x^2}{a}N$ 3) $\frac{3.6x}{a^3}N$ 4) $\frac{4.8x}{a^2}N$

KEY : 3

Hint : Conceptual

105. **Three identical charges each $2\mu C$ lie at the verticles of a right angled triangle as shown the figure. Forces on the charge at B due to the charges at A and C respectively are F_1F_2. The angle between their resultant force and F_2.**



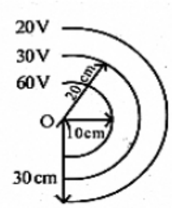
1) $\tan^{-1}\left(\frac{9}{16}\right)$ 2) $\tan^{-1}\left(\frac{9}{7}\right)$ 3) $\tan^{-1}\left(\frac{16}{9}\right)$ 4) $\tan^{-1}\left(\frac{7}{8}\right)$

KEY : 3

Hint : $Tan\alpha = \frac{F_1}{F_2}$

106. **The figure shows equipotential surfaces concentric at 'O'. The magnitude of electric at a distance 'r' meters from 'O' is**

1) $\frac{9}{r^2}Vm^{-1}$ 2) $\frac{16}{r^2}Vm^{-1}$ 3) $\frac{2}{r^2}Vm^{-1}$ 4) $\frac{6}{r^2}Vm^{-1}$



KEY : 4

Hint : $E = -\frac{dv}{dr}$

107. A region contains a uniform electric field $\overline{E} = \left(10\hat{i} + 30\hat{j}\right)Vm^{-1}$. A and B are two points int eh field at (1,2,0) and (2,1,3)m respectively. The work done when a charge of 0.8C moves from A to B in a parabolic path is

1) 8J 2) 80J 3) 40J 4) 16J

KEY : 4

Hint : $W = vq \,\&\, V = -\int \overline{E}.\overline{dr}$

108. When a long straight uniform rod is connected across an ideal cell, the drift velocity of electron in it is v. if a uniform hole is made along the axis of the rod and the same battery is used, then the drift velocity electrons becomes.

1) v 2) >v 3) <v 4) zero

KEY : 1

Hint : Conceptual

109. In a meter bridge experiment, when a nichrome wire is in the right gap, the balancing length is 60cm. When the nichrome wire is uniformly stretched to increase its length by 20% and again connected in the right gap, the new balancing length is nearly.

1) 61cm 2) 31cm 3) 51cm 40 41cm

KEY : 3

Hint : $\frac{X}{R} = \frac{l_1}{100 - l_1}$

110. A loup of flexible conducting wire lies in a magnetic field of 2.0 T with its P perpendicular to the field. The length of the wire is 1m. When a current of 1.1A is pa through the loop. It opens into a circle, then the tension developed in the wire is

1) 0.15N 2) 0.25N 3) 0.35N 4) 0.45N

KEY: 3

Hint : Conceptual

111. A charge q is spread uniformly over an isolated ring of radius 'R'. The ring is rotated about its natural axis with an angular velocity 'ω'. Magnetic dipole moment of the ring is

(1) $\frac{q\omega R^2}{2}$ (2) $\frac{q\omega R}{2}$ (3) $q\omega R^2$ (4) $\frac{q\omega}{2R}$

KEY: 1

HINT: $M = iA = \frac{qv}{2\pi r} \times \pi r^2$

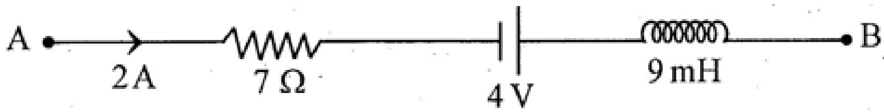
112. A magnetic dipole of moment $2.5\,Am^2$ is free to rotate about a vertical axis passing through its centre. It is released from East-West direction. Its kinetic energy at the moment is takes North-South position is $\left(B_H = 3\times10^{-5}T\right)$.

(1) 50 μJ (2) 100 μJ (3) 175 μJ (4) 75 μJ

KEY: 4

HINT: Gain in KE = loss in PE = MB

113. A branch of a circuit is shown in the figure. If current is decreasing at the rate of $10^3\,As^{-1}$, the potential difference between A and B is



(1) 1 V (2) 5 V (3) 10 V (4) 2 V

KEY: 1

HINT: $V_A - 2\times 7 + 4 - 9\times 10^{-3}\left(-10^{3}\right) = V_B$

114. The natural frequency of an LC circuit is 125 kHz. When the capacitor is totally filled with a dielectric material, the natural frequency decreases by 25 kHz. Dielectric constant of the material is nearly

(1) 3.33 (2) 2.12 (3) 1.56 (4) 1.91

KEY: 3

HINT: $\frac{f_1}{f_2} = \sqrt{K} \Rightarrow K = \left(\frac{f_1}{f_2}\right)^2$

115. Choose the correct sequence of the radiation sources in increasing order of the wavelength of electromagnetic waves produced by them.

(1) X-ray tube, Magnetron valve, Radio active source, Sodium lamp
(2) Radio active source, X-ray tube, Sodium lamp, Magnetron valve
(3) X-ray tube, Magnetron valve, Sodium lamp, Radio active source
(4) Magnetron valve, Sodium lamp, X-ray tube, Radio active source

KEY: 2

HINT: Conceptual

116. A photo sensitive metallic surface emits electrons when X-rays of wavelength 'λ' fall on it. The de Broglie wavelength of the emitted electrons is (Neglect the work function of the surface, m is mass of the electron. H-Planck's constant, c-velocity of light)

(1) $\sqrt{\frac{2mc}{h\lambda}}$ (2) $\sqrt{\frac{h\lambda}{2mc}}$ (3) $\sqrt{\frac{mc}{h\lambda}}$ (4) $\sqrt{\frac{h\lambda}{mc}}$

KEY: 2

HINT: $\frac{hc}{\lambda} = KE = \frac{P^2}{2m} \Rightarrow P = \sqrt{\frac{2mhc}{\lambda}}$

$\lambda_m = \frac{h}{P} = \sqrt{\frac{h\lambda}{2mc}}$

117. An electron in a hydrogen atom undergoes a transition from a higher energy level to a lower energy level. The incorrect statement of the following is

(1) Kinetic energy of the electron increases
(2) Velocity of the electron increases
(3) Angular momentum of the electron remains constant
(4) Wavelength of de-Broglie wave associated with the motion of electron decreases

KEY: 3

HINT: Conceptual

118. The radius of germanium (Ge) nuclide is measured be twice the radius of ${}^{9}_{4}Be$. The number of nucleons in Ge will be

(1) 72 (2) 73 (3) 74 (4) 75

KEY: 1

HINT: $R \propto A^{1/3} \Rightarrow \frac{R_2}{R_1} = \left(\frac{A_2}{A_1}\right)^{1/3} \Rightarrow 2^3 = \frac{A_2}{9}$

119. For a common-emitter transistor amplifier, the current gain is 60. If the emitter current is 6.6 mA then its base current is

(1) 6.492 mA (2) 0.108 mA (3) 4.208 mA (4) 0.343 mA

KEY: 2

HINT: $\frac{I_E}{I_B} = 1+\beta \Rightarrow I_B = \frac{I_E}{1+\beta}$

120. If a transmitting antenna of height 105m is placed on a hill, then its coverage area is

(1) $4224\,km^2$ (2) $3264\,km^2$ (3) $6400\,km^2$ (4) $4864\,km^2$

KEY: 1

HINT: $A = \pi d^2 = \pi(2Rh_T)$

CHEMSITRY

121. In which of the following, the product of uncertainity in velocity and uncertainity in position of a micro particle of mass 'm' is not less than

1) $h \times \frac{3\pi}{m}$ **2)** $\frac{h}{3\pi} \times m$ **3)** $\frac{h}{4\pi} \times \frac{1}{m}$ **4)** $\frac{h}{4\pi} \times m$

KEY: 3

HINT: $m\Delta v.\Delta n \geq \frac{h}{4\pi} \Rightarrow \quad \Delta v.\Delta n \geq \frac{h}{4\pi} \times \frac{1}{m}$

122. An element has $[Ar]3d^1$ configuration in its +2 oxidation state. Its position in the periodic table is

1) period-3, group-3 **2) period-3, group-7**
3) period-4, group-3 **4) period-3, group-9**

KEY: 3

HINT: $4s^2 3d^1 = Scandium$

Period: 4, group:3

123. In which of the following molecules all bond lengths are not equal?

1) SF_6 **2)** PCl_5 **3)** BCl_3 **4)** CCl_4

KEY: 2

HINT: $PCl_5 \rightarrow 2arial$

3 Equitorial

Arial>equatorial

124. In which of the following molecules maximum number of lone pairs is present on the central atom?

1) NH_3 **2)** H_2O **3)** ClF_3 **4)** XeF_2

KEY: 4

HINT: In XeF_2

2 bond pairs , 3 lone pairs

125. Which one of the following is the kinetic energy of a gaseous mixture containing 3g of hydrogen and 80g of oxygen at temperature T(K)?

1) 3RT **2) 6RT** **3) 4RT** **4) 8RT**

KEY: 2

HINT: $n = \frac{3}{2} + \frac{80}{32} = 4$

$KE = \frac{3}{2}nRT = \frac{3}{2} \times 4RT = 6RT$

126. A, B, C and D are four different gases with critical temperatures 304.1, 154.3, 405.5 and 126.0K respectively. While cooling the gas which gets liquefied first is

1) B **2) A** **3) D** **4) C**

KEY: 4

HINT: High critical temperature

$\Rightarrow$ easily liquificable

127. 40ml of $\underline{x}$ M $KMnO_4$ solution is required to react completely with 200ml of 0.02M oxalic acid solution in acidic medium. The value of $\underline{x}$ is

1) 0.04 **2) 0.01** **3) 0.03** **4) 0.02**

KEY: 1

HINT: $N_1V_1 = N_2V_2$

$40 \times 5 \times x = 200 \times 0.02 \times 2$

$\Rightarrow x = 0.04$

128. Given that

$C_{(s)} + O_{2(g)} \rightarrow CO_{2(g)}; \Delta H^o = -x\,kJ$

$2CO_{(g)} + O_{2(g)} \rightarrow 2CO_{2(g)}; \Delta H^o = -y\,kJ$

The enthalpy of formation of CO will be

1) $\frac{y-2x}{3}$ **2)** $\frac{y-2x}{2}$ **3)** $\frac{2x-y}{2}$ **4)** $\frac{x-y}{2}$

KEY: 2

HINT: $Eq(1) - \frac{Eq(2)}{2} \quad \Rightarrow \frac{y-2x}{2}$

129. At 400K, in a 1.0L vessel N_2O_4 is allowed to attain equilibrium

$N_2O_{4(g)} \rightleftharpoons 2NO_{2(g)}$

At equilibrium the total pressure is 600mm Hg, when 20% of N_2O_4 is dissolved. The K_p value for the reaction is

1) 50 **2) 100** **3) 150** **4) 200**

KEY: 2

HINT: $N_2O_4 \rightleftharpoons 2NO_2$

C O

$\frac{4C}{5}$ $\frac{2C}{5}$

$\frac{4C}{5} + \frac{2C}{5} = \frac{6C}{5} = 600$

$\Rightarrow C = 500$

$K_p = \frac{\left(\frac{2C}{5}\right)^2}{\frac{4C}{5}} = \frac{C}{5} = \frac{500}{5} = 100$

130. In which of the following salts only cationic hydrolysis is involved?

1) CH_3COONH_4 2) CH_3COONa 3) NH_4Cl 4) Na_2SO_4

KEY: 3

HINT: cationic hydrolysis

S.A.+W.B.

$\therefore NH_4Cl$

$NH_3 + HCl$

$W.B. \quad S.A.$

131. Calgon is

(1) Na_2HPO_4 **(2)** Na_3PO_4 **(3)** $Na_6P_6O_{18}$ **(4)** NaH_2PO_4

KEY: 3

SOL: $Na_6P_6O_{18}$

132. Consider the following statements.

I) Cs^+ ion is more highly hydrated than other alkali metal ions

II) Among the alkali metals only lithium forms a stable nitride by direct combination with nitrogen

III) Among alkali metals Li, Na, K, Rb, the metal Rb has the highest melting point

IV) Among alkali metals Li, Na, K, Rb only Li forms peroxide when heated with oxygen

(1) I (2) II (3) III (4) IV

KEY: 2

SOL: Due to more Hydration energy

133. Assertion (A) : $AlCl_3$ exists as a dimer through halogen bridged bonds.

Reason (R) : $AlCl_3$ gets stability by accepting electrons from the bridged halogen

(1) Both (A) and (R) are true and (R) is the correct explanation of (A)

(2) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(3) (A) is true, but (R) is not true

(4) (A) is not true, but (R) is true

KEY: 1

SOL: Due to completion of octet

134. Which of the following causes "Blue baby syndrome"

(1) High concentration of lead in drinking water

(2) High concentration of sulphates in drinking water

(3) High concentration of nitrates in drinking water

(4) High concentration of copper in drinking water

KEY: 3

SOL: High concentration of Nitrate

135. Which of the following belongs to the homologous series of $C_5H_8O_2N$

1) $C_6H_{10}O_3N$ 2) $C_6H_8O_2N_2$ 3) $C_6H_{10}O_2N_2$ 4) $C_6H_{10}O_2N$

KEY: 4

SOL: due to difference of CH_2

136. In Dumas method, 0.3g of an organic compound gave 45mL if nitrogen at STP. The percentage of nitrtogen is

1) 16.9 2) 18.7 3) 23.2 4) 29.6

KEY: 2

SOL: $\%N = \frac{28}{22400} \text{x} \frac{45}{0.3} \text{x} 100 = 18.75\%$

137. The IUPAC name of

$(CH_3)_2CH-CH=CH-CH=CH-\underset{C_2H_5}{CH}-CH-CH_3$ **is**

1) 2, 7-dimethyl -3, 5-nonadiene
2) 2, 7-dimehtyl -2-ehtylheptadiene
3) 2-methyl -7 –ethyl-3, 5-octadiene
4) 1, 1-dimethyl-6-ethyl-2, 4-heptadiene

KEY: 1

SOL: $CH_3-\underset{CH_3}{CH}-CH=CH-CH=CH-CH-\underset{CH_3}{CH}-CH_2-CH_3$

138. Match the following

LIST – I	**LIST-II**
A) Ferromagnetic	**1)** O_2
B) Anti ferro magnetic	**2)** CrO_2
C) Ferri magnetic	**3)** MnO
D) Para magnetic	**4)** Fe_3O_4
	5) C_6H_6

	A	B	C	D
(1)	3	2	4	1
(2)	2	3	4	1
(3)	1	3	5	4
(4)	4	2	3	5

KEY: 2

SOL: conceptual

139. The vapour pressures of pure benzene and toluene are 160 and 60mm Hg respectively. The mole fraction of benzene is vapour phase incontact with equimolar solution of benzene and toluene is

1) 0.073 2) 0.027 3) 0.27 4) 0.73

KEY: 4

SOL: $P_T = 160\,x\,\frac{1}{2} + 60\,x\,\frac{1}{2} = 110$ $\quad P_B = P_B^O\,x\,B = 160\,x\,\frac{1}{2}$

$P_B = P_T X_B \Rightarrow 80 = 110.Y_B$

$Y_B = \frac{80}{110} = 0.73$

140. 6g of a non volatile , non electrolyte

(1) 60 (2) 140 (3) 180 (4) 120

KEY: 4

SOL: $\Delta T_f = K_t m$

$0.93 = 1.86\,x\,\frac{6}{GMW}\,x\,\frac{1000}{100} \Rightarrow GMW = 120$

141. The products obtained at the cathode and anode respectively during the electrolysis of aqueous K_2SO_4 **solution using platinum electrodes are**

1) O_2, H_2 2) H_2, O_2 3) H_2, SO_2 4) K, SO_4

KEY : 2

Hint : Conceptual

142. The slope of the graph drawn between /nk and 1/T as per Arrhenius equation gives the value (R = gas constant, Ea = Activation energy)

1) $\frac{R}{E_a}$ 2) $\frac{E_a}{R}$ 3) $\frac{-E_a}{R}$ 4) $\frac{-R}{E_a}$

KEY : 3
Hint : Conceptual

143. Which is not the correct statement in respect of chemisorptions?
1) Highly specific adsorption 2) Irreversible adsorption
3) Multilayered adsorption 4) High enthalpy of adsorption

KEY : 3
Hint : Conceptual

144. Which of the following is carbonate ore?
1) Cuprite 2) Sideritc 3) Zineite 4) Bauxite

KEY : 2
Hint : $FeCO_3$

145. Which one of the following statement is not correct?
1) O_3 is used as germicide
2) In O_3, O-O bond length is identical with that of molecular oxygen
3) O_3 is an oxidizing agent
4) The shape of O3 molecule is angular

KEY : 2
Hint : Conceptual

146. Which of the following reactions does not take place?
1) $F_2 + 2Cl \rightarrow 2F^- + Cl_2$ 2) $Br_2 + 2l^- \rightarrow 2Br + l_2$
3) $Cl_2 + 2Br \rightarrow 2Cl^- + Br_2$ 4) $Cl_2 + 2F^- \rightarrow 2Cl + F_2$

KEY : 4
Hint : Cl_2 cannot displace F_2

147. Which of the following statements regarding sulphur is not correct?
1) At about 1000K, it mainly consists of S2 molecules
2) The oxidation state of sulphur is never less than +4 in its compounds
3) S2 molecule is paramagnetic
4) Rhombic sulphur is readily soluble in CS2

KEY : 2
Hint : Conceptual

148. Which of the following reactions does not involve, liberation of oxygen?
1) $XeF_4 + H_2O \rightarrow$ 2) $XeF_4 + O_2F_2 \rightarrow$ 3) $XeF_2 + H_2O_2 \rightarrow$ **4)** $XeF_6 + H_2O \rightarrow$

KEY : 3
Hint : $X_eF_6 + 3H_2 \rightarrow FeO_3 + 6HF$

149. Select the correct IUPAC name
1) Penta ammonia carbonate cobalt (III) chloride 2) Pentammine carbonate cobalt chloride
3) Pentammine carbonato cobalt (III) chloride 4) Cobalt (III) pentammine carbonate chloride

KEY : 3
Hint : Conceptual

150. Which of the following characteristics of the transition metals is associated with their catalytic activity?
1) Color of hydrated ions 2) Diamagnetic behavior
3) Paramagnetic behavior 4) Variable oxidation

KEY : 4
Hint : Conceptual

151. Observe the following polymers

PHBV	**Nylon 2-nylon 6**	**Giyptal**	**Bakelite**
(A)	**(B)**	**(C)**	**(D)**

(1) (D) (2) (A), (B) (3) (D) (4) (C), (D)

KEY: 2

HINT: Conceptual

152. Observe the following statements

i) Sucrose has glycosidic linkage

ii) Cellulose is present in both plants and animals

iii) Lactose contains D-galactose and D-glucose units

(1) (i), (ii), (iii) (2) (i), (ii) (3) (ii), (iii) (4) (i), (iii)

KEY: 4

HINT: Conceptual

153. Identify the antioxidant used in foods

(1) Aspartame (2) Sodium benzoate

(3) Ortho-sulpho benzimide (4) Butylated hydroxyl toluene

KEY: 4

HINT: Conceptual

154.

This reaction is know as

(1) Wurtz-Fittig reaction (2) Wurtz reaction

(3) Fittig reaction ($) Friedel-crafts reaction

KEY: 1

HINT:

Reaction between alkal halide and aryl halide with sodium metal in the presence of dry ether

155. What is *Z* in the following sequence of reactions?

2-methyl-2-bromo propane $\xrightarrow[dry\ ether]{Mg} X \xrightarrow{H_2O} Z$

(1) Propane (2) 2-methyl propene (3) 2-methyl propane (4) 2-methyyl butane

KEY: 3

HINT:

2-methyl-2-bromo propane $\xrightarrow[dry\ ether]{Mg} X \xrightarrow{H_2O} Z$

$$X = CH_3-\overset{\overset{MgBr}{|}}{\underset{\underset{CH_3}{|}}{C}}-CH_3 \qquad Y = CH_3CH\left(CH_3\right)CH_3$$

156. In which of the following reactions the product is <u>not</u> correct?

(1) $CH_3CHO \xrightarrow{LiAlH_4} CH_3CH_2OH$

(2) $CH_3COCH_3 \xrightarrow[HCl]{Zn-Hg} CH_3-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_3$

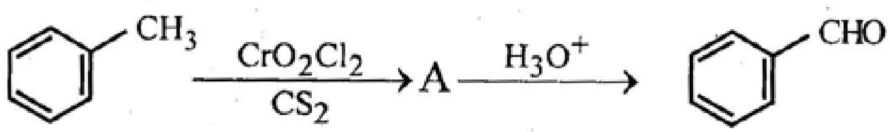
(3) $CH_3CH_2CHO \xrightarrow[(ii)\,KOH,\,ethylene\ glycol/\Delta]{(i)\,H_2N-NH_2} CH_3CH_2CH_3$

(4) $CH_3CH_2CHO \xrightarrow{KMnO_4} CH_3CH_2COOH$

KEY: 2

HINT: $CH_3COCH_3 \xrightarrow[HCl]{Zn-Hg} CH_3-CH_2-CH_3$

157. Identify the name of the following reaction



(1) Gatterman – Koch reaction (2) Gatterman reaction

(3) Stephen reaction (4) Etard reaction

KEY: 4

HINT: Conceptual

158. What is C in the following sequence of reactions?

$CH_3OH \xrightarrow{PCl_3} A \xrightarrow{KCN} B \xrightarrow{hydrolysis} C$

(1) CH_3CH_2OH (2) CH_3CHO (3) CH_3COOH (4) $HOCH_2-CH_2OH$

KEY: 3

HINT:

$CH_3OH \xrightarrow{PCl_3} A \xrightarrow{KCN} B \xrightarrow{hydrolysis} C$

$A = CH_3Cl,\ B = CH_3CN,\ C = CH_3COOH$

159. The order of basic strength of the following in aqueous solution is

$C_6H_5NH_2$	$(CH_3)_3N$	NH_3	CH_3NH_2	$(CH_3)_2NH$
1	2	3	4	5

(1) 4 > 1 > 5 > 3 > 2 (2) 2 > 5 > 4 > 3 > 1

(3) 5 > 4 > 2 > 3 > 1 (4) 4 > 3 > 5 > 2 > 1

KEY: 3

HINT: Conceptual

160. Yellow dye can be prepared by a coupling reaction of benzene diazonium chloride in acid medium with X. Identify X from the following

(1) Aniline (2) Phenol (3) Cumene (4) Benzene

KEY: 1

HINT: Benzene diazonium chloride undergoes coupling with aniline in acidic medium